Multi-Dimensional DSP

Final Project

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# Project Introduction

Direction of arrival (DOA) estimation is well known problem in many real-life applications in fields such as communications and radar. Along with the growing need in such applications as part of new technologies such as 5G & IOT communications, the need of good estimation algorithms of multiple spatial TF (Time-Frequency) signals is growing accordingly.

The following project will present the recent work of the following paper: “An Efficient Direction of Arrival Estimation Algorithm for Sources with Intersecting Signature in the Time–Frequency Domain” – by Nabeel Ali Khan, Sadiq Ali and Kwonhue Choi [[1](#Ref_1)].

Moreover, the project contains: Summary of similar methods presented, extra simulations results (along with attached simulations files), possible flaws and points to pay attention in the main article algorithm, and suggestions for improvements/innovations and supporting simulations for those propositions.

# Keywords

Time-Frequency; Crossing components; direction of arrival estimation; instantaneous frequency (IF); STFD;

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# Problem Introduction

Direction of arrival (DOA) is a well-known problem as presented in the main introduction, yet there are open issues regarding the formerly known solutions.

Graphical user interface, application

Description automatically generatedProblem set-up:

Figure 1. Uniform linear array receiving signal from multiple sources.

The signal model shown in [[Fig\_1](#Fig_1)] and defined in [[1](#Ref_1), [2](#Ref_2)]:



Where **s**(t) is the signals vector, **w**(t) is the noise vector and **A** is the channel matrix represents the connection between the Uniform linear array (ULA) antennas array set-up to the signal received in every antenna. The matrix composed of the column vectors:

The signals assumed to be Frequency Modulated (FM) signals, and we wish to extract the angles shown in [[Fig\_1](#Fig_1)] of the FM signals.

We strive to get an estimation that:

1. Will give good results for non-stationary signals that can be intersect each other.
2. Will be efficient as possible.
3. Works for over-determined (e.g., #signals < #antennas) and under-determined problems (e.g., #signals >= #antennas).
4. Will has less assumptions as possible on the signal model (except that it is FM signals).

A few solutions presented in the following sections, with emphasis on solutions proposed by the authors of [[1](#Ref_1)]. Further improvements and suggestions for solutions are proposed and shown later in sections [!!!!!!!!!].

# Former Work Summary

In this section we first show related work for a brief background, then summary the work presented in the introduction to help understand the algorithm proposed [[1](#Ref_1)].

## Related Work

### TF MUSIC (Multiple Signal Classification)

The TF-MUSIC algorithm for DOA [[2](#Ref_2)] is an enhancement of the widely used MUSIC algorithm, that allows us to estimate signals that might be non-stationary. The MUSIC algorithm first computes the covariance matrix of the received signals and then performs decomposition of the covariance matrix to estimate the DOA.

Instead of this known method, the TF-MUSIC uses the STFD which is a generalization of the TFD to a vector signal. The advantage is that the TFD concentrating the energy of the signal components at their IF (Instantaneous frequency) curves while the noise energy spreads in the entire TF plane.

For the signal model defined in ([1](#eq_1)) We define the STFD matrix as:

Where m and f represent the time index and the frequency index, respectively. The kernel characterizes the TFD and is a function of both the time and the lag variables (m,l).

After an SVD decomposition we get:

Where **Es** and **En** spans the signal subspace and the noise subspace and where **D**ss is the signal TFD matrix.  
Usually we could simply use SVD to estimate **Es** and **En**, except for the case where **D**ss is singular (for that there are other solutions such as a joint block-diagonalization (JBD)).

Once we estimated the noise subspace we peak the N largest values of the localization function:

MATLAB package for TF applications such as mentioned above was developed later and used in this current paper by [[3](#Ref_3)].

### SADTFD (Spatial Adaptive Directional TFD) based on Viterbi DOA estimation

The following solution presented in [[4](#Ref_4),[5](#Ref_5)] first takes the STFD of the signal, then enhances the STFD with directional smoothing to an Adaptive directional TFD (ADTFD) and employs the Viterbi algorithm for IF estimation followed by source localization.

In the first step mentioned the signal is transformed to the TF domain using quadratic TFD with the Wigner–Ville distribution (which is energy concentration ideal for mono-component linearly FM signals). After that, we average the STFD over all sensors to reduce some cross-TFDs components.   
Since we have multiple signals, a low-pass filtering is applied using a smoothing kernel . To avoid loss in auto-terms, an adaptive smoothing kernel is used, and its main property is that the filter response is maximum when aligned along ridges and reduced to 0 as it becomes orthogonal to ridges.  
So in each point is rotated along the ridges to suppress the cross-terms.

Computational complexity: This SADTFD step has cost, where V is total number of sources, M is number of sensors, Ns is number of samples in time domain and L is the smoothing filter length.

In the next step the estimated IF is used to iteratively remove the corresponding component from the mixture signal.   
In order to find a signal path in the TF domain, there is an optimization problem developed in [[5](#Ref_5)] which is more robust to noise. The IF estimation and TF filtering method in steps:

1. In order to estimate one IF component for the k-th signal:
   1. At each step we look at the cost at the n-1 step (which represents the previous time sample), and computes the cost of transition to the frequency bin j from the frequency bin k in the n-1 step:

Where is the cost of the partial best path for the bin k at time instant

n – 1, and g, h and q are penalty functions described in the paper.

* 1. The optimal path for the j-th bin is chosen by taking .
  2. The optimal path leading to the TF point (n, j) is obtained by merging (n, j) with the path: and now we obtain the cost for current j-th bin in the n step: .
  3. The algorithm continues recursively backward N steps, and the path which minimizes corresponds to the IF of the strongest component.

1. Now a TF filtering is performed:
   1. The Viterbi algorithm from previous section estimates the IF of the kth component, e.g., . Then the instantaneous phase is estimated by:
2. 1. The signal is de-chirped to a new signal: under the assumption that so we get (under neglecting noise):
4. The signal is now estimated by applying a low-pass filter, and a time domain MUSIC algorithm is applied on the estimated signal.
5. The synthesized signal from previous section is removed from the current signal.
6. Now iteratively repeat the above steps for the next signal until done.

### DOA based on IF estimation using ridge tracking

This solution uses similar ADTFD method as mentioned in the first step of the [SADTFD](#_SADTFD_(Spatial_Adaptive) algorithm (except for the fact that here it is noted that only auto-terms in the STFD are calculated in order to reduce complexity to ), where variables are same as in [SADTFD](#_SADTFD_(Spatial_Adaptive_1) complexity section).

In the next step the algorithm performs blind source separation using TF filtering at each sensor which is less complexed than the previously mentioned Viterbi algorithm.

After TF analysis, a blind separation is performed in the following steps as detailed in [[6](#Ref_6)]:

1. A point of highest energy is detected in a TF plane.
2. The IF of kth component at time instant is estimated as:  
    is the estimated IF at time instant .

The IF for is estimated using the following procedure:

1. Increment , where Ts is sampling time interval.
2. Detect peak in the neighbourhood of and define: , where is the ADTFD in time t, and frequency f.
3. The detected peak may belong to IF curve of any other component in case of intersecting components. In order to assign the peak to appropriate IF curve, the next peak detected will be taken if its direction is close enough, otherwise an extrapolation will be taken:
4. Repeat step 3 and forward in both directions of the signal until done.

After done with the IF estimation the signals are de-chirped and estimated as in section 2 in [SADTFD](#_SADTFD_(Spatial_Adaptive_1) , and for the signals estimated we apply time domain MUSIC for each signal.

## Fast-IF Main article summary

### Introduction

In [[1](#Ref_1)] a new algorithm is proposed for the DOA estimation that is based on fast IF estimation combined with time domain MUSIC algorithm.

The main solutions used to comparison in the article presented in [Related Work](#_Related_Work_1) section above.

\* More solutions mentioned in the article such as: connected component linking, blind source separation-based methods, morphological image processing techniques to extract TF signatures and parametric Hough transform (which gives solution to similar problem but under Linear frequency model assumption). Those methods will not be discussed here.

While the TF MUSIC algorithm is only applicable for over-determined problems (e.g., number of sources < number of antennas), the other two methods (developed and presented by former works of the same authors) are good also for both under and over determined problems.  
although those methods work well according to the authors, they are computationally expensive since they both require adaptive TFD which has high computational complexity is shown in [SADTFD](#_SADTFD_(Spatial_Adaptive_1). We now present the more efficient proposed algorithm.

The Fast-IF algorithm overview

The IF estimation is based on finding slow variations in the signal energy in the TF domain:

1. The energy within a short interval {t-∆T, t+∆T} is found and averaged across the M sensors for the kth sensor:

(10)

1. We obtain the highest energy time instant .
2. A Fractional Fourier Gaussian window (FRFT) which is a generalization of Fourier transform. It has an adjustable parameter in the form of (in our case) rotational angle that makes it more useful in the TF domain. The window presented in [[1](#Ref_1)] is used to window segments of the signals.

Fourier transform of the windowed signal at kth sensor:

1. The fourier transform is averaged across the M-sensors to reduce noise and then the optimum frequency and rotation orders are selected according to:
2. Starting from t0 , the IF is estimated for the case t>t0 (and the same way for t<t0).   
    is updated as: , and then the IF at estimated by the f0 and α that maximize the spatially averaged correlations of multi-sensor signals with the TF-shifted analysis window:

(13)

The search is restricted around {f0 − ∆f, f0 + ∆f} and around {α0 − ∆α0, α0+∆α0} where ∆α0 = 1/(2L+1) in order to reduce computational complexity.

1. Now when done extracting the IF component, de-chirping of the signals is performed followed by a time domain MUSIC process as showed in [de-chirping](#de_chirping) section and we get our angles of DOA for each signal source.

### Illustrations

To illustrate the algorithm work we will show spectrogram of the input signal from one Sensor of two intersecting signals, and the spectrogram of the signals estimated:



ss



Figure 3. Spectrogram illustration of the IF separation

A picture containing text

Description automatically generatedThe Fast-IF algorithm mechanism can be illustrated as in [[Fig\_3](#Fig_2)] taken from the main paper [[1](#Ref_1)].

Figure 3. Illustration of the Fast-IF algorithm for DOA estimation

### Computational Complexity

The details about the following calculations can be found in the papers mentioned so far. Here we will present a comparison of the complexity for the different algorithms:

|  |  |  |  |
| --- | --- | --- | --- |
| Algorithm  Step | [SADTFD](#_SADTFD_(Spatial_Adaptive_1) + Viterbi | [Ridge tracking IF est.](#_DOA_based_on) | [Fast-IF MUSIC](#_The_Fast-IF_MUSIC) |
| STFD calculation |  |  | - |
| TF filtering /  IF estimation |  |  |  |
| Peak (energy) | - | - |  |
| FRFT | - | - |  |
| Total |  |  | )) |

\* Where W is the cost of computing FRFT at a single bin, P is number of bins searched over in every step ([step 5](#step_5_fast_if)), LF is number of Fourier transform operations and all other variables are as in [SADTFD](#_SADTFD_(Spatial_Adaptive_1) complexity section.  
\* MUSIC algorithm cost is ignored as it’s applied to all (and even negligible).

Notes:

* Since we have many variables in the complexity calculation, we will take an example to show the differences in numbers:  
  Let us have 2 signals (V) and array of 3 antennas (M). Assuming we have 128 samples (Ns) and we are using 32x32 (LxL) directional smoothing filter. The number of bins searched over (P) is determined by the quantization factor L=100 ([step 3](#step_3_fast_if)). The window length of the FRFT (W) selected to be 32. From the window size chosen we get 4 Fourier transform operations (LF). All in all, we get:

In conclusion, we see that the Fast-IF cost is the smallest for reasonable parameters chosen.

* Even though we saw that the Fast-IF is better than the SADTFD in order of magnitude (since the Viterbi algorithm is computationally heavy), it is not the case regarding to comparison with the complexity of the ridge tracking (since the cross-terms calculation is more efficient then the SADTFD case).

# Further Results

The papers results can be found in the original papers and were checked using the original code in the [Original GitHub Link](https://github.com/nabeelalikhan1/DOA-using-FAST-IF). Instead, in the following section we will present results using new simulations in order to examine the performance of the algorithms mentioned (particularly the Fast-IF) in the [Former Work Summary](#_Former_Work_Summary), and also to get new insights about the algorithm flaws for future improvements.

## Gain Differences Effect

Let us consider the two signals coming from -15° and 15°, that are received by three sensors in two cases: crossing signals in TF domain and non-crossing signals in TF domain.

Let us examine two problems, one for crossing signal and other for non-crossing signals:

For non-crossing signals scenario, we define:

.

For crossing signals scenario, we define:

.

Where g is a normalization factor that examines gain differences between two signals.

First, we will show the DOA and IF estimation of the signals for the Fast-IF algorithm with g=1/10:

Figure 4. Non-crossing signals - Fast-IF signals and DOA estimations with gain difference

  
Figure 5. Crossing signals - Fast-IF signals and DOA estimations with gain difference

In the above figures we can see the following problem: the first (and stronger) signal captured by the Fast-IF estimation fits well with the real one, but after de chirping the estimation of the next IF component does not fit. As a result, the DOA for the 1st signal is good but the 2nd DOA far from the original one.

To further understand when the algorithm fails for gain differences a simulation of mean error in degrees under very high SNR for each signal estimated was performed:

1. (b)

Figure 6. Mean absolute error in degrees for varying differences in gains   
(a) Non-crossing signals, (b) Crossing signals

We see that in the crossing signals scenario the problem is much more prominent, and gain differences (even of 10dB and more) are a real life scenarios and hence it is a weakness of the algorithm.

Note 1: We define the mean absolute error as:

Where is the angle of the kth source in the nth trial (and so on the calculation is done for every scenario of gain difference for example).

Note 2: The same tests were performed for the other DOA estimation method mentioned in the [Related Work](#_Related_Work_1), and same problem was found in all of them.

The above-mentioned issues are a reasonable result of the algorithm, yet it points out problems in the Fast-IF method:

1. The Fast-IF method works with a known number of signal sources, yet it is not always known, and in cases where the signal is weak perhaps it is not a real signal but an interference.
2. The Fast-IF method is strongly based on version of the power spectrum (with FRFT instead regular Fourier Transform), and since one of the signals is weak, after de-chirping there is perhaps still enough energy from the first signal. As a result, the IF estimation seems to have problems in the initial frequency peak.  
   We do notice that the general curve direction of the IF is somewhat detected by the Fast-IF.

# Possible improvements & Innovations

In the following section we will discuss the problems arises from the [Further Results](#_Further_Results) section and present a small new improvement for the current IF estimation.  
Also we will discuss and show an innovation in the usage of the algorithms mentioned for a 2D DOA estimation (estimation of angles in horizontal and vertical axes).

## Unknown number of sources for FAST-IF

It was mentioned in the [Further Results](#_Further_Results) that the algorithm is assuming to know the number of sources, as a result of this assumption, the FAST-IF always founds a given number of signals even when one of the signals might be just some weak interference signal and not a wanted one.

It is also suggesting that the algorithm is not suitable for unknown number of sources.

Yet, there is a possible simple solution that should achieve nice results (under assumptions that detailed in the following sections).

The proposed solution:

1. We first assume we have at least one signal, then take the energy detected after calculating the energy that maximizes the spatially averaged correlations (as calculated in the [Fast-IF introduction](#_Introduction) section 5).
2. Calculate an averaged mean over the 2L steps defined in the formula from section 1 to slightly reduce effects of false peaks in the FRFT calculation when SNR is low.   
   The mathematical formula derived:
3. We set our threshold now to be the TH = (Max Energy) \* (Relative TH), where we perform test to find optimal Relative TH. Now we continue as usual with the algorithm.
4. Continue with the algorithm including de-chirping the previous signal until calculating again the Max Energy using the equation from section (2) above.
5. Compare the current energy detected to the TH calculated in (3). If the energy is lower stop and move on to (6), else, raise the number of sources in 1, and go back to 3.
6. Compute the DOA using the MUSIC algorithm as usual for the IF estimations. Check if the peak value of the MUSIC power spectrum achieved is bigger then a threshold. If the peak is smaller than the threshold, the result considered not valid, then stop and reduce number of estimations to the number of valid MUSIC results found so far.

The proposed solution only slightly changes the original algorithm, and hence have almost zero computational complexity.  
We do note that we take in to account two assumptions:

1. There is at least one signal transmitted.
2. The number of sources is smaller/equal to the number of sensors.

The algorithm modified according to the suggested improvement and a test to find the optimal threshold for the inner number of IF estimations under high SNR, then we compare under varying SNR the performance degradation using the simple new detection and IF estimation method proposed.

The results of the tests for finding relative threshold for the number of IF estimations (section (3) of the above proposed solution) presented in [Figure 7](#Fig_6).

(b)

(a)

Figure 7. Results of detection error using IF estimation threshold only  
 (a) Scanning varying thresholds , (b) Error Rate VS SNR for optimal threshold

From the [Figure 7](#Fig_6) graph (a) we derived the relative threshold of 0.1, and now since we see that the new adaptive algorithm does not achieve zero error rate in detection of the number of sources, we of course expected for some degradation in the performance against a scenario where the number of signals is known.  
In [Figure 7](#Fig_6) (b) we see that the detection improvement suggested is tend to the optimal lowest detection error rate in SNR > ~20dB, but achieve disappointing results for lower SNR.

As a result of this bad results in low SNR, step 6 was added since we see that the over-estimation rate is the dominant detection error.

In [Figure 8](#Fig_7) we can see results for using the full steps of the proposed detection solution:

Note 1: Even though we have relatively high error rate in the lower SNR region, we do notice that the absolute average error in every SNR is approximately 1, which means that when the algorithm does a mistake, it misses or overestimating by approximately only one signal.

The computational complexity of the new algorithm is of course larger for cases where the SNR is low since we tend to perform number of IF estimations as the number of the sensors (V=M in the [complexity table](#complexity_table)). On the other hand, we can “implement” the algorithm in a way where after every IF estimation, a MUSIC algorithm is calculated and compared to the threshold, and by this way reduce the complexity overhead to only one more calculation of IF estimation + MUSIC in most of the cases. Under high SNR we saw that in most cases the computational complexity overhead is negligible.



Figure 8. Results of detection error combining threshold on MUSIC and IF estimation

## 2D DOA Estimation

### 2D Estimation concept

So far, we saw how we can find the DOA of our sources using angle of arrival in the plain of our ULA (Uniform Linear Array). Yet today many times we will be interested in finding 2D DOA using an elevation angle ( and azimuth angle ( which help us for example in applications of beamforming in the direction of signals we are interested in.

For this purpose, we offer an L-shaped antennas array that will simplify the transition to 2D estimation since the arrays are orthogonal.

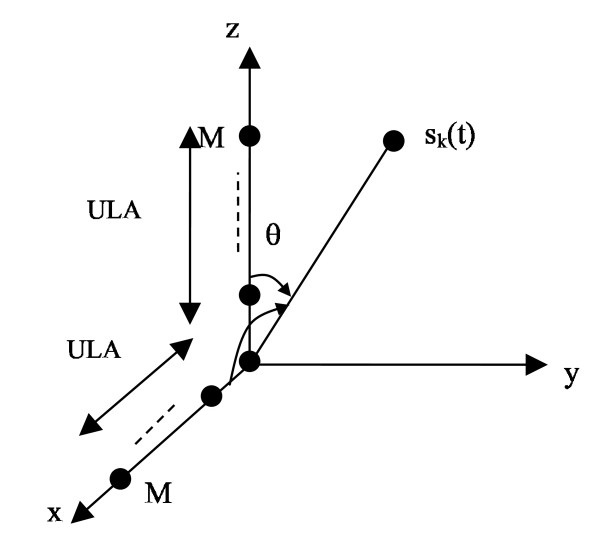


Figure 9. L-shaped antennas array for 2D DOA estimation

In this new set up of the problem, we can define the steering vectors for the z-axis and x-axis with the help of [[7](#Ref_7)], and get the following representation:

(17)

(18)

The algorithm we purpose for 2D DOA estimation based on the Fast-IF method is as follows:

1. Calculate instantaneous frequency for the z antennas array and x antennas array separately using the Fast-IF method presented in [Fast-IF Main article summary](#_Fast-IF_Main_article).
2. After calculating the IF for the orthogonal plains presented in [figure 9](#Fig_8) above, we apply MUSIC algorithm separately for each plain to find the angles of arrival: .
3. Since we are interested in the azimuth angle we can use a few geometrical transitions using the fact that we can calculate the x-y plain triangle sides as follows:

Now using the law of cosine, we can derive the following connection:

And derive our estimated using the inverse cosine function (which is injective for 0-180 degrees).

Note: A use of 2D MUSIC algorithm was examined in order to avoid using the MUSIC algorithm twice for each source, yet it isn’t efficient in this case since calculating the 2D MUSIC invariant requires calculation of the noise matrices components (instead of noise vector components in the 1D case) and hence not efficient in comparison to using 1D MUSIC twice since the arrays are orthogonal and the calculation for each axis separately is simple.

We now show proof of concept using the simulation to show one source DOA in the plain:

 Figure 9. 2D DOA estimation versus reference for one signal

Figure 10. 2D DOA estimation versus reference for three signals

# Appendix

## Matlab overview

The new Matlab simulations are based on previous simulations from [[1](#Ref_1)], and including some adaptations, and some new simulations.

All simulations are attached in <https://github.com/oriohayon/MDDSP-project.git> .

New simulations:

* Adaptive\_fast\_if\_test – new simulation for examining results and finding thresholds for the new detection ability suggested for the algorithm (includes the MUSIC threshold found for second stage threshold in the detection capability suggested).
* Adaptive\_sources\_FAST\_IF – the FAST IF algorithm script including new code for implementing part of the new detection ability.
* Create\_FM\_Signals – a function that creates different kinds of chirp signals for different purposes.
* DOA\_MAE simulations – testing the Mean Average Error metric for new results (shows more clearly the average angle deviation).
* FAST\_IF\_2D\_DOA – the 2-dimensional implementation of the algorithm.
* DOA\_Plots\_2D – script for the 2D plots.

Rest of the simulations are derived from the original repository link in [[1](#Ref_1)].

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